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Implementation of Cooperative Dynamic Behaviors in Networked Systems with Controllability Preservation

Lorenzo Sabattini[1], Cristian Secchi[1], Matteo Cocetti[2] and Cesare Fantuzzi[1]

Abstract— **Decentralized control of networked systems has been widely investigated in the literature, with the aim of obtaining coordinated emerging behaviors (e.g. synchronization, swarming, coverage, formation control) by means of local interaction. In this paper we consider the possibility of injecting external inputs into the networked system, in order to obtain more complex cooperative behaviors. Specifically, modeling the dynamics of the networked system as a standard LTI system, we introduce a strategy to ensure preservation of the controllability property of the networked system, under time varying topologies. Then, we introduce a strategy that makes it possible to control the overall state of the networked system by directly controlling only a subset of the networked agents, namely the leaders. Exploiting local interaction rules, it is possible to define the leaders inputs in such a way that each follower is forced to track a desired periodic setpoint.**

I. INTRODUCTION

This paper introduces a methodology to implement dynamic complex behaviors in a networked system. The main objective is to have a subset of agents, called *leaders*, that are in charge of controlling the overall state of the networked system, in a completely decentralized manner.

Decentralized control of networked systems has been widely addressed in the last few years. Main application fields include, for instance, multi–robot systems [1], distributed sensor networks [2] and interconnected manufacturing equipments [3].

Generally speaking, the aim of decentralized control strategies is implementing local interaction rules to obtain a coordinated emerging behavior. Mainly investigated coordinated behaviors include aggregation, swarming, formation control, coverage and synchronization [1], [4]–[6].

The idea of implementing more complex cooperative behaviors have recently appeared in the literature. For instance, [7], [8] present decentralized strategies for the coordination of groups of mobile robots moving along non–trivial paths. A decentralized strategy is presented in [9] that extends the standard consensus protocol to obtain periodic geometric patterns.

Recently a few works appeared that investigate the possibility of *interacting* with a networked system, in order to obtain a desired behavior [10]. The idea is that of having a set of agents, interconnected by means of a graph: a subset of those agents, namely the *leaders*, may be directly controlled, while the others, namely the *followers*, are indirectly controlled through the underlying interconnection graph.

As shown in [11], it is possible to model a networked system in such a way that the classical notions of controllability and observability of LTI systems are applicable. Specifically, it turns out that in the case of networked systems these properties are heavily influenced by the topology of the underlying communication graph. Several works can be found in the literature that develop strategies to infer the controllability property of networked systems. For instance, [12], [13] provide sufficient conditions on the graph topology to verify the controllability property. On the same lines, the work presented in [14] provides necessary and sufficient conditions on the spectrum of the Laplacian matrix, to verify the controllability of a networked system.

When weighted graphs are considered, the concept of structural controllability can be exploited. Considering a linear system defined by a generic structure, and a set of variable parameters, the concept of *structural controllability* [15], [16] refers to the possibility of ensuring the controllability property with a particular choice of the parameters. This concept has been recently applied to networked systems [17]–[19] interconnected by means of an edge–weighted graph. Specifically, structural controllability of a weighted graph identifies the possibility of making a networked system controllable with an opportune choice of the edge–weights. As shown in [19], it is possible to demonstrate that a networked system is structurally controllable if and only if the underlying graph is connected. The control strategy introduced in [20] guarantees, in a completely decentralized fashion, the connectivity of the networked system almost surely. This strategy is based on a connectivity maintenance algorithm first introduced in [21], and on the choice of random edge–weights.

Duality principle can be invoked to show that a networked system is controllable if and only if it is observable. Once observability and controllability have been guaranteed, then classical feedback control law design strategies can be extended to networked systems, in order to obtain the desired cooperative behavior.

As shown in [22], the well known *regulator equations* can be exploited to design a control law that makes a networked system follow a predefined periodic setpoint.

In this paper we initially describe a methodology, first introduced in [20], to ensure the preservation of the controllability property for networked systems, in a decentralized manner. Subsequently, the control strategy first introduced

[1]L. Sabattini, C. Secchi and C. Fantuzzi are with the Department of Science and Methods for Engineering (DISMI), University of Modena and Reggio Emilia, Italy {lorenzo.sabattini, cristian.secchi, cesare.fantuzzi}@unimore.it
[2]M. Cocetti is with the University of Trento, Italy matteo.cocetti@gmail.com

in [22] will be described, to make the networked system follow a predefined periodic setpoint function.

The paper is organized as follows. The model of the system is introduced in Section II. Then, Section III describes a strategy for decentralized controllability (and observability) preservation. In Section IV a technique is described to produce a desired periodic setpoint, exploiting an exosystem. The control law is then defined in Section V, to make each follower track the desired setpoint, by means of an input injected into the leaders. Finally, Section VII contains some concluding remarks.

II. Preliminaries

A. Background on graph theory

Let $\mathscr{G}$ indicate a generic undirected graph: throughout the paper we will always refer to undirected graphs, unless otherwise specified. Let $\mathsf{V}(\mathscr{G})$ and $\mathsf{E}(\mathscr{G})$ be the vertex set and the edge set of the graph $\mathscr{G}$, respectively. Moreover, let N be the cardinality of $\mathsf{V}(\mathscr{G})$ (i.e. the number of vertices, or nodes, of the graph), and let M be the cardinality of $\mathsf{E}(\mathscr{G})$ (i.e. the number of edges, or links, of the graph). Clearly, $\mathsf{E} \subseteq \mathsf{V} \times \mathsf{V}$.

Let $v_i, v_j \in \mathsf{V}(\mathscr{G})$ be the i–th and the j–th vertices of the graph, respectively. Then, v_i and v_j are neighbors if $(v_i, v_j) \in \mathsf{E}(\mathscr{G})$. Given an undirected graph, $(v_i, v_j) \in \mathsf{E}(\mathscr{G})$ if and only if $(v_j, v_i) \in \mathsf{E}(\mathscr{G})$.

Let $\mathcal{L}(\mathscr{G})$ be the unweighted Laplacian matrix, and let $\mathcal{L}_{\mathcal{W}}(\mathscr{G})$ be the weighted Laplacian matrix.

B. Model of the system

Consider a group of N agents, namely mobile robots, sensors or other entities, whose interconnection structure is modeled by means of an undirected graph $\mathscr{G}$. Let $x_i \in \mathbb{R}^m$ be the state of the i–th agent: without loss of generality, we will hereafter consider the case where the state corresponds to each agent's position. Then, let the agents be interconnected according to the well known (weighted) consensus protocol [6]:

$$\dot{x}_i = -w_{ij} \sum_{j \in \mathsf{N}_i} (x_i - x_j) \tag{1}$$

where $w_{ij} > 0$ is the edge weight, and $\mathsf{N}_i \subseteq \mathsf{V}(\mathscr{G})$ is the neighborhood of the i–th agent, defined as the set of the agents that are interconnected to the i–th one, namely:

$$\mathsf{N}_i = \{ j \in \mathsf{V}(\mathscr{G}) \text{ such that } (v_i, v_j) \in \mathsf{E}(\mathscr{G}) \} \tag{2}$$

Without loss of generality, we will hereafter refer to the scalar case, namely $x_i \in \mathbb{R}$. It is however possible to extend all the results to the multi–dimensional case, considering each component independently.

Hence, let $x = [x_1, \dots, x_N]^T \in \mathbb{R}^N$ be the state of the multi–agent system. The interaction rule defined in Eq. (1) can be rewritten as follows:

$$\dot{x} = -\mathcal{L}_{\mathcal{W}}(\mathscr{G})\, x \tag{3}$$

As is well known [6], under the consensus protocol the states of the agents converge to a common value. Assume now that the goal is to control the states of the networked agents: for this purpose, define a few leader agents, to whom it is possible to inject a control action. The state of the other agents, referred to as the followers, evolves according to the consensus protocol.

More specifically, let $\mathsf{V}_L(\mathscr{G}) \subset \mathsf{V}(\mathscr{G})$ be the set of the leader agents, and let $\mathsf{V}_F(\mathscr{G}) = \mathsf{V}(\mathscr{G}) - \mathsf{V}_L(\mathscr{G})$ be the set of the follower agents. Then, as shown in [11] for unweighted graphs, the interaction rule introduced in Eq. (1) is modified as follows:

$$\begin{cases} \dot{x}_i = -w_{ij} \sum\limits_{j \in \mathsf{N}_i} (x_i - x_j) & \text{if } v_i \in \mathsf{V}_F(\mathscr{G}) \\ x_i = u_i & \text{if } v_i \in \mathsf{V}_L(\mathscr{G}) \end{cases} \tag{4}$$

where $u_i = u_i(t) \in \mathbb{R}$ is a control input.

Let N_L be the number of leaders. It is always possible to index the agents such that the last N_L agents are the leaders, and the first $N_F = N - N_L$ are the followers. Then, as shown in [11], it is possible to decompose the Laplacian matrix $\mathcal{L}_{\mathcal{W}}(\mathscr{G})$ as follows:

$$\mathcal{L}_{\mathcal{W}}(\mathscr{G}) = -\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{B}^T & \mathcal{L}_L \end{array} \right] \tag{5}$$

where $\mathcal{A} = \mathcal{A}^T \in \mathbb{R}^{N_F \times N_F}$ is the Laplacian matrix of the subgraph of the followers, $\mathcal{B} \in \mathbb{R}^{N_F \times N_L}$ represents the interconnection among leaders and followers, and $\mathcal{L}_L = \mathcal{L}_L^T \in \mathbb{R}^{N_L \times N_L}$ is the Laplacian matrix of the subgraph of the leaders.

Define now $x_F \in \mathbb{R}^{N_F}$ as the state vector of the followers, namely $x_F = [x_1, \dots, x_{N_F}]^T$. Define also $u \in \mathbb{R}^{N_L}$ as the input vector, namely $u = [u_{N_F+1}, \dots, u_N]^T$. Moreover, let $y \in \mathbb{R}^{N_L}$ be the output vector, that is the vector containing the state variables that are measurable by the leaders: it is reasonable to assume that each leader is able to measure the state of its neighbors. The overall closed loop system can then be represented as in Fig. 1.

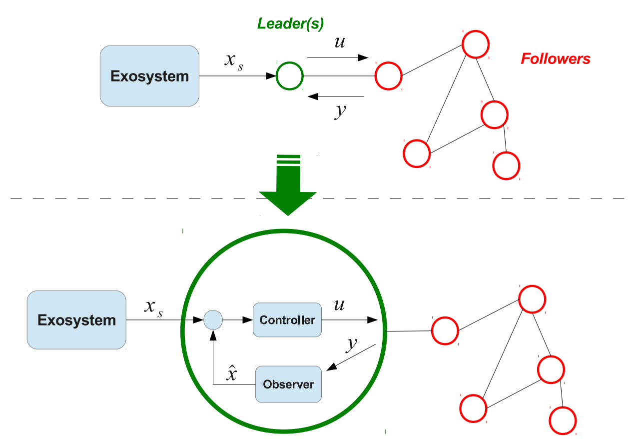

Fig. 1. Closed loop system scheme

We assume that the leader nodes are able to directly exchange information among each other. Namely, the following assumption is made:

Assumption 1 *A complete communication graph exists among the leader nodes.*

Therefore, the output vector can be defined as the vector containing the states of each leader's neighbors. Namely, the dynamics of the networked system can then be rewritten as follows:

$$\begin{cases} \dot{x}_F &= \mathcal{A}x_F + \mathcal{B}u \\ y &= \mathcal{B}^T x_F \end{cases} \tag{6}$$

III. CONTROLLABILITY AND OBSERVABILITY

The dynamics of the networked system have then been rewritten in Eq. (6) as a standard LTI system. Hence, the classical notions of controllability and observability can be applied to the networked system itself. In particular, the following property can be derived:

Property 1 *A networked system whose dynamics are written according to Eq.* (6) *is observable if and only if it is controllable.*

In fact, as is well known, controllability property is related to the matrices $\mathcal{A}$ and $\mathcal{B}$. Namely, the controllability matrix $\mathcal{C}$ is defined as follows:

$$\mathcal{C} = \left[\mathcal{B} \,|\, \mathcal{AB} \,|\, \mathcal{A}^2\mathcal{B} \,|\, \ldots \,|\, \mathcal{A}^{N_F-1}\mathcal{B}\right] \tag{7}$$

According to the definition given in Eq. (6), the observability matrix $\mathcal{O}$ is given by:

$$\mathcal{O} = \mathcal{C}^T \tag{8}$$

A LTI system is controllable if and only if its controllability matrix is full rank, and it is observable if and only if its observability matrix is full rank. According to the definition of the LTI representation of the networked system given in Eq. (5), and as shown in Eq. (8), a networked system is controllable if and only if it is observable.

We will hereafter describe a methodology to ensure the preservation of the controllability property in a networked system, in a decentralized manner.

For this purpose, we introduced the following Lemma, that was derived in [14].

Lemma 1 [14, Theorem 5] *Consider a networked system whose dynamics are described as in Eq.* (3). *Then, the networked system is controllable if and only if the Laplacian matrix $\mathcal{L}_{\mathcal{W}}(\mathscr{G})$ has exactly N distinct eigenvalues.*

In this paper we will show how to exploit edge weights to ensure the controllability in a decentralized manner.

For this purpose, we exploit the concept of structural controllability, that has been widely addressed in [15], [16], [19]. The structural controllability property was defined as follows:

Definition 1 (Structural controllability, [19])
A networked system is said to be structurally controllable *if and only if it can be made controllable by means of an appropriate choice of the edge weights.*

In [19] authors demonstrated that structural controllability is strongly related to the connectivity of the underlying graph. Specifically, the following Lemma was derived:

Lemma 2 [19, Theorem 1] *A networked system is structurally controllable if and only if the underlying graph is connected.*

A. Structural controllability preservation

In this Section we introduce a decentralized strategy to ensure the preservation of the structural controllability property for the networked system.

The proposed strategy relies Lemma 2: specifically, we will exploit a connectivity maintenance control strategy, in order to ensure structural controllability preservation.

Let $\mathcal{C}(\mathscr{G}) \in \mathbb{R}^{M \times M}$ be the communication edge weight matrix. This matrix is defined as $\mathcal{C}(\mathscr{G}) = \operatorname{diag}(c)$, where $c \in \mathbb{R}^M$ will be defined subsequently as a vector of weights related to the communication constraints in the networked system.

Consider then the weighed Laplacian matrix $\mathcal{L}_{\mathcal{C}}(\mathscr{G})$. In [21], [23], the following kinematic model was considered:

$$\dot{x}_i = u_i^{\mathcal{C}} \tag{9}$$

with the control law $u_i^{\mathcal{C}}$ defined as follows:

$$u_i^{\mathcal{C}} = -\frac{\partial V\left(\lambda_2\left(\mathcal{L}_{\mathcal{C}}(\mathscr{G})\right)\right)}{\partial x_i} \tag{10}$$

where the energy function $V\left(\lambda_2\left(\mathcal{L}_{\mathcal{C}}(\mathscr{G})\right)\right)$ will be defined hereafter. The aim of this control law is to ensure that, given a desired threshold $\epsilon > 0$, then the value of the algebraic connectivity does never go below this threshold, as the system evolves. For this purpose, the energy function $V\left(\lambda_2\left(\mathcal{L}_{\mathcal{C}}(\mathscr{G})\right)\right)$ is chosen according to the following definition.

Definition 2 (Energy Function)
An energy function $V(\lambda_2(\cdot)) : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is defined such that the following properties hold:

1) *It is continuously differentiable $\forall \lambda_2(\cdot) > \epsilon$.*
2) *It is non–negative.*
3) *It is non–increasing with respect to $\lambda_2(\cdot)$, $\forall \lambda_2(\cdot) \geq \epsilon$.*
4) *It approaches a constant value, as $\lambda_2(\cdot)$ increases.*
5) $\lim_{\lambda_2(\cdot) \mapsto \epsilon} V(\lambda_2(\cdot)) = \infty$
6) $\lim_{\lambda_2(\cdot) \mapsto \epsilon} \left\| \frac{\partial V(\lambda_2(\cdot))}{\partial \lambda_2(\cdot)} \right\| = \infty$

In [21], the energy function was defined as follows:

$$V\left(\lambda_2\left(\mathcal{L}_{\mathcal{C}}(\mathscr{G})\right)\right) = \coth\left(\lambda_2\left(\mathcal{L}_{\mathcal{C}}(\mathscr{G})\right) - \epsilon\right) \tag{11}$$

Let R be the maximum communication range for each agent, i.e. the j–th agent is inside N_i if $\|x_i - x_j\| \leq R$. Then, the vector c is defined as the collection of the edge–weights c_{ij}, defined as follows:

$$c_{ij} = \begin{cases} e^{-\left(\|x_i - x_j\|^2\right)/\left(2\kappa^2\right)} & \text{if } \|x_i - x_j\| \leq R \\ 0 & otherwise \end{cases} \tag{12}$$

The scalar parameter κ is chosen to satisfy the threshold condition $e^{-\left(R^2\right)/\left(2\kappa^2\right)} = \Delta$, where Δ is a small predefined threshold. This definition of the edge–weights introduces a discontinuity in the control action, that can be avoided introducing a smooth bump function, as in [24].

This definition of the edge–weights is motivated by the fact that $\lambda_2(\cdot)$ is a non–increasing function of each edge–weight [25]: hence, as two connected robots increase their distance, the value of $\lambda_2(\cdot)$ decreases, until they disconnect.

It is worth noting that, even though the algebraic connectivity of the communication graph is a global quantity, the connectivity maintenance control action can be implemented exploiting an estimate of $\lambda_2(\cdot)$. Specifically, a bounded–error decentralized estimation procedure was introduced in [21] for the estimation of $\lambda_2(\cdot)$.

The dynamics of the networked system introduced in Eq. (4) can then be extended as follows:

$$\begin{cases} \dot{x}_i = -w_{ij} \sum\limits_{j\in\mathsf{N}_i} (x_i - x_j) + u_i^{\mathcal{C}} & \text{if } v_i \in \mathsf{V}_F(\mathscr{G}) \\ \\ \dot{x}_i = \dot{u}_i + u_i^{\mathcal{C}} & \text{if } v_i \in \mathsf{V}_L(\mathscr{G}) \end{cases} \tag{13}$$

with the control law $u_i^{\mathcal{C}}$ defined according to Eq. (10), $\forall v_i \in \mathsf{V}(\mathscr{G})$, and the control law u_i is defined as in Eq. (4).

Theorem 1 *Consider the dynamical system described in Eq.* (13)*, and let the networked system be structurally controllable at time $t = 0$. Then, the control strategy in Eq.* (10) *ensures the preservation of the structural controllability property.*

Proof: Proof is based on the fact that the energy function does not increase as the system evolves. Details can be found in [20]. ■

It is worth remarking that, according to the Property 4 in Definition 2, the control action $u_i^{\mathcal{C}}$ vanishes at steady state, namely when the algebraic connectivity of the graph is sufficiently big. For this reason, we will hereafter consider the dynamics of the system described in Eqs. (4) and (6), assuming that the control action $u_i^{\mathcal{C}}$ becomes non–negligible only when, due to the relative displacements of the agents, it becomes necessary to enforce the structural controllability property.

B. Decentralized controllability of the network

In Section III-A we defined a control strategy that ensures preservation of the structural controllability property. According to Definition 1, given a structurally controllable graph, a choice of edge weights exists such that the corresponding weighted graph is controllable.

Hence, assuming the graph is structurally controllable, in this Section we will define a completely decentralized strategy to define the edge weights in order to ensure the controllability of the graph.

For this purpose, the following definition of *almost sure controllability* was introduced in [20]:

Definition 3 (Almost sure controllability)
A LTI system is almost surely controllable *if the corresponding controllability matrix has full rank with probability one.*

The following Lemma describes a property of random matrices that will be exploited for the proof of the following Theorem.

Lemma 3 [26, Theorem 2.5.2] *The set of symmetric random matrices with distinct eigenvalues is of full Lebesgue measure.*

It is then possible to show that a random choice of the edge weights leads to almost sure controllability.

Theorem 2 *Consider the dynamical system described in Eqs.* (4) *and* (6)*, and assume that the underlying graph $\mathscr{G}$ is structurally controllable. Then, assigning a Gaussian random weight $w_{ij} \in \mathsf{G}$ to each edge $(v_i, v_j) \in \mathsf{E}(\mathscr{G})$ ensures almost sure controllability of the corresponding weighted graph.*

Proof: Proof is based on the fact that, according to Lemma 3, a random choice of the edge weights ensures that $\mathcal{L}_{\mathcal{W}}(\mathscr{G})$ has, with probability one, exactly N distinct eigenvalues. According to Lemma 1, this implies that the networked system is controllable. Details can be found in [20]. ■

Hence, as shown in Section III-A, structural controllability can be ensured in a completely decentralized manner. Subsequently, almost sure controllability can be enforced setting random weights on the edges of the graph. In order to maintain an undirected graph structure, it is necessary for each couple of neighboring agents to agree on the same (random) value for the weight of the edge between them. This can be obtained in several manners: for instance, they may independently compute two random values, and use the average as the edge weight. It is worth remarking that, in order to preserve the properties of the Laplacian matrices summarized in Section II-A, it is necessary to ensure that each edge weight is a positive value.

C. State observer

Hence, we will hereafter suppose that the (possibly weighted) communication graph $\mathscr{G}$ is designed in such a way that the corresponding LTI system defined as in Eq. (6) is controllable and observable.

Therefore, once observability is guaranteed, a standard Luenberger state observer [27] can be designed. Specifically, let $\hat{x} \in \mathbb{R}^{N_F}$ be the estimate of x_F, and let $\mathcal{K}_l \in \mathbb{R}^{N_F \times N_L}$ be an opportunely chosen gain matrix. The following update law may be defined for the state observer:

$$\dot{\hat{x}} = \mathcal{A}\hat{x} + \mathcal{B}u - \mathcal{K}_l\left(y - \mathcal{B}^T\hat{x}\right) \tag{14}$$

It is worth remarking that, under Assumption 1, this state observer can be implemented in a decentralized manner, that is exploiting only information locally available to the leaders.

Let the estimation error $\hat{e} \in \mathbb{R}^{N_F}$ be defined as

$$\hat{e} = x_F - \hat{x} \tag{15}$$

The estimation error converges to zero if and only if the eigenvalues of the matrix $\left(\mathcal{A}+\mathcal{K}_l\mathcal{B}^T\right)$ are all negative. As is well known, if the system is observable, then it is always possible to find a matrix $\mathcal{K}_l$ that satisfies this condition.

IV. Periodic setpoint definition

In this Section we will define an exosystem [28], whose state is exploited to generate a desired periodic setpoint for the networked system. More specifically, we will show how to generate a setpoint that approximates a generic periodic function as the linear combination of $n \in \mathbb{N}$ harmonics.

For this purpose, consider a generic periodic function $f(t) \in \mathbb{R}$, with period $T > 0$, that is

$$f(t) = f(t+T) \ \ \forall t \in \mathbb{R} \tag{16}$$

Define then the exosystem as an autonomous system whose state vector $\xi \in \mathbb{R}^{2n+1}$ evolves according to the following dynamics:

$$\dot{\xi} = \mathcal{G}\xi \tag{17}$$

where the matrix $\mathcal{G} \in \mathbb{R}^{(2n+1)\times(2n+1)}$ is a block diagonal matrix, namely:

$$\mathcal{G} = \begin{bmatrix} \tilde{\mathcal{G}}_0 & 0 & \dots & 0 \\ 0 & \tilde{\mathcal{G}}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \tilde{\mathcal{G}}_n \end{bmatrix} \tag{18}$$

Let each block $\tilde{\mathcal{G}}_p$ be defined as follows:

$$\tilde{\mathcal{G}}_p = \begin{cases} 0 & \text{if } p = 0 \\ \begin{bmatrix} 0 & p\dfrac{2\pi}{T} \\ -p\dfrac{2\pi}{T} & 0 \end{bmatrix} & \text{if } p = 1, \dots, n \end{cases} \tag{19}$$

According to the definition given in Eqs. (17), (18) and (19), it is possible to show that the exosystem exhibits the following property:

Property 2 *Any periodic function with period T may be approximated with a linear combination of the elements of the vector ξ.*

Consider then a generic periodic setpoint $x_s(t) \in \mathbb{R}^{N_F}$, that has to be tracked by the state $x_F(t) \in \mathbb{R}^{N_F}$ of the followers. According to Property 2, it is possible to define a matrix $\mathcal{J} \in \mathbb{R}^{N_F\times(2n+1)}$ such that

$$x_s(t) = \mathcal{J}\xi(t) \tag{20}$$

Therefore, once the period T and the number of harmonics n have been defined, then the choice of the matrix $\mathcal{J}$ leads to the definition of a particular periodic setpoint x_s.

V. Definition of the control law for setpoint tracking

In this Section we introduce a methodology to define a control law that makes the followers track a periodic setpoint defined according to Eq. (20). For this purpose, let the regulation error $e(t) \in \mathbb{R}^{N_F}$ be defined as follows:

$$e(t) = x_F(t) - \mathcal{J}\xi(t) \tag{21}$$

Then, considering Eqs. (6), (17) and (21), the dynamics of the networked system can be rewritten as follows:

$$\begin{cases} \dot{x}_F &= \mathcal{A}x_F + \mathcal{B}u \\ y &= \mathcal{B}^T x_F \\ e &= x_F - \mathcal{J}\xi \\ \dot{\xi} &= \mathcal{G}\xi \end{cases} \tag{22}$$

As is well known [28] the regulation problem can be solved defining the input u as follows:

$$u = \mathcal{F}x_F + (\Gamma - \mathcal{F}\Pi)\,\xi \tag{23}$$

where $\mathcal{F}$ is an arbitrary matrix, chosen such that $(\mathcal{A}+\mathcal{B}\mathcal{F})$ is Hurwitz stable, and Π and Γ are the solution of the *regulator equations* that, considering the dynamical system described in Eq. (22), can be written as follows:

$$\begin{cases} \mathcal{A}\Pi + \mathcal{B}\Gamma = \Pi\mathcal{G} & \text{(24a)} \\ \Pi - \mathcal{J} = 0 & \text{(24b)} \end{cases}$$

It is worth noting that Eq. (24a) defines a generalized Sylvester equation [29], while Eq. (24b) can be solved satisfying the following equality:

$$\mathcal{J} = \Pi \tag{25}$$

This equality represents a constraint on matrix $\mathcal{J}$, that, according to Eq. (20), implies a constraint on the choice of the setpoint.

Considering then the definition of the setpoint functions $x_s(t)$ given in Eq. (20), and considering the regulator equations in Eq. (24), we now introduce the following definition of *admissible setpoint functions*.

Definition 4 *The set of admissible setpoint functions $\mathsf{S}_a \in \mathbb{R}^{N_F}$ is defined as follows:*

$$\mathsf{S}_a = \{x_s(t) = \mathcal{J}\xi(t) \text{ such that } \mathcal{J} = \Pi\} \tag{26}$$

where Π is a solution of the generalized Sylvester equation in Eq. (24a)*, namely*

$$\exists \Gamma \text{ such that } (\Pi, \Gamma) \text{ satisfy Eq. (24a)}$$

Theorem 3 *Consider the definition of the setpoint functions given in Eq.* (20)*, and consider the definition of the admissible setpoint functions S_a given in Eq.* (26)*. Then, given the topology of the graph, a matrix $\mathcal{H}$ can be univocally defined such that its image defines the the set S_a.*

Proof: Proof is omitted due to space limitations. Details can be found in [22]. ■

The generalized Sylvester equation defined in Eq. (24a) can then be exploited to define a control law to make the system follow an admissible setpoint, following the methodology described in [29, Theorem 1].

Therefore, the control law in Eq. (23) makes the networked system follow the desired setpoint function, defined as in

Eq. (20), where matrix $\mathcal{J}$ is defined in order to satisfy the conditions of Theorem 3.

However, for the control law to be applicable in a decentralized manner, the vector x_F in Eq. (23) has to be replaced with its estimate, computed by means of the state observer introduced in Eq. (14).

Considering the dynamics of the networked system defined in Eq. (6), the estimation error $\hat{e}$ defined in Eq. (15), the state observer defined in Eq. (14), and the control law defined in Eq. (23), the dynamics of the closed loop system can then be summarized as follows (Fig. 1):

$$\begin{bmatrix} \dot{x}_F \\ \dot{\hat{e}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathcal{A}+\mathcal{B}\mathcal{F} & -\mathcal{B}\mathcal{F} & \mathcal{B}(\Gamma - \mathcal{F}\Pi) \\ \mathbb{O} & \mathcal{A}+\mathcal{K}_l\mathcal{B}^T & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathcal{G} \end{bmatrix} \begin{bmatrix} x_F \\ \hat{e} \\ \xi \end{bmatrix} \tag{27}$$

where $\mathbb{O}$ is a zero matrix of opportune dimension.

VI. Simulations

For validation purposed, simulations were performed with Matlab, and on simulated groups of quadrotor UAVs. Simulations are summarized in a video clip, that is freely available online[1].

VII. Conclusions

In this paper we introduced a methodology to obtain complex cooperative behaviors in networked systems. Modeling the dynamics of the networked system as a standard LTI system, we introduced a strategy to ensure preservation of the controllability property of the networked system, under time varying topologies.

Then, we introduced a control strategy that makes it possible to control the overall state of the networked system by directly controlling only a subset of the networked agents, namely the leaders. Assuming a fixed communication topology, we demonstrated that it is possible to exploit standard design methodologies, based on the regulator equations, to make each follower agent track a desired periodic setpoint function. Current work aims at extending the scope of the proposed control strategy to variable topology communication graphs, thus explicitly including the controllability preservation technique in the control law design.

[1] http://www.arscontrol.unimore.it/icra13ws

References

[1] L. Sabattini, C. Secchi, and C. Fantuzzi, "Arbitrarily shaped formations of mobile robots: artificial potential fields and coordinate transformation," *Autonomous Robots (Springer)*, vol. 30, no. 4, pp. 385–397, may 2011.

[2] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *Communications Magazine, IEEE*, vol. 40, no. 8, pp. 102–114, 2002.

[3] P. R. Wurman, R. D'Andrea, and M. Mountz, "Coordinating hundreds of cooperative, autonomous vehicles in warehouses," *AI Magazine*, vol. 29, no. 1, pp. 9–20, 2008.

[4] A. Ganguli, J. Cortes, and F. Bullo, "Multirobot rendezvous with visibility sensors in nonconvex environments," *Robotics, IEEE Transactions on*, vol. 25, no. 2, pp. 340 –352, april 2009.

[5] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," 2004.

[6] R. Olfati–Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi–agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[7] L. C. A. Pimenta, G. A. S. Pereira, N. Michael, R. C. Mesquita, M. M. Bosque, L. Chaimowicz, and V. Kumar, "Swarm coordination based on smoothed particle hydrodynamics technique," *Robotics, IEEE Transactions on*, vol. PP, no. 99, pp. 1 –17, 2013.

[8] L. Sabattini, C. Secchi, and C. Fantuzzi, "Closed–curve path tracking for decentralized systems of multiple mobile robots," *Journal of Intelligent and Robotic Systems (Springer)*, 2012 (in press).

[9] P. Tsiotras and L. Castro, "Extended multi-agent consensus protocols for the generation of geometric patterns in the plane," in *American Control Conference (ACC), 2011*, 29 2011-july 1 2011, pp. 3850 – 3855.

[10] A. F. A., C. Secchi, H. I. Son, H. H. Bulthoff, and P. Robuffo Giordano, "Bilateral teleoperation of groups of mobile robots with time-varying topology," *Robotics, IEEE Transactions on*, vol. 28, no. 5, pp. 1019–1033, 2012.

[11] M. Egerstedt, S. Martini, M. Cao, K. Camlibel, and A. Bicchi, "Interacting with networks: How does structure relate to controllability in single-leader, consensus networks?" *Control Systems, IEEE*, vol. 32, no. 4, pp. 66 –73, aug. 2012.

[12] M. Ji and M. Egersted, "A graph-theoretic characterization of controllability for multi-agent systems," in *American Control Conference, 2007. ACC '07*, july 2007, pp. 4588 –4593.

[13] A. Rahmani, M. Ji, M. Mesbahi, and M. Egerstedt, "Controllability of multi-agent systems from a graph-theoretic perspective," *SIAM Journal on Control and Optimization*, vol. 48, no. 1, pp. 162–186, Feb. 2009.

[14] M. Franceschelli, S. Martini, M. Egerstedt, A. Bicchi, and A. Giua, "Observability and controllability verification in multi-agent systems through decentralized laplacian spectrum estimation," in *Decision and Control (CDC), 2010 49th IEEE Conference on*, dec. 2010, pp. 5775 –5780.

[15] C. T. Lin, "Structural controllability," *Automatic Control, IEEE Transactions on*, vol. 19, no. 3, pp. 201 – 208, jun 1974.

[16] R. Shields and J. Pearson, "Structural controllability of multiinput linear systems," *Automatic Control, IEEE Transactions on*, vol. 21, no. 2, pp. 203 – 212, apr 1976.

[17] S. Sundaram and C. N. Hadjicostis, "Structural controllability and observability of linear systems over finite fields with applications to multi-agent systems," *Automatic Control, IEEE Transactions on*, vol. 58, no. 1, pp. 60 –73, jan. 2013.

[18] F. Sorrentino, "Effects of the network structural properties on its controllability," *Chaos*, vol. 17, no. 3, 2007.

[19] M. Zamani and H. Lin, "Structural controllability of multi-agent systems," in *American Control Conference, 2009. ACC '09.*, june 2009, pp. 5743 –5748.

[20] L. Sabattini, C. Secchi, and C. Fantuzzi, "Controllability preservation for networked systems with time varying topologies," in *IEEE Conference on Decision and Control (submitted)*, 2013.

[21] L. Sabattini, N. Chopra, and C. Secchi, "On decentralized connectivity maintenance for mobile robotic systems," in *Proceedings of the IEEE Conference on Decision and Control*, 2011.

[22] M. Cocetti, L. Sabattini, C. Secchi, and C. Fantuzzi, "Decentralized control strategy for the implementation of cooperative dynamic behaviors in networked systems," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (submitted)*, 2013.

[23] L. Sabattini, N. Chopra, and C. Secchi, "Distributed control of multi–robot systems with global connectivity maintenance," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2011.

[24] K. D. Do, "Formation tracking control of unicycle–type mobile robots with limited sensing ranges," *IEEE Transactions on Control Systems Technology*, vol. 16, pp. 527–538, 2008.

[25] C. Godsil and G. Royle, *Algebraic Graph Theory*. Springer, 2001.

[26] G. W. Anderson, A. Guionnet, and O. Zeitouni, *An Introduction to Random Matrices*. Cambridge Press, 2009.

[27] E. D. Sontag, *Mathematical Control Theory: Deterministic Finite Dimensional Systems*. Springer, 1998.

[28] A. Saberi, A. A. Stoorvogel, and P. Sannuti, *Control of Linear Systems with Regulation and Input Constraints*. Springer-Verlag New York, LLC, 2000.

[29] A. G. Wu, G. R. Duan, and B. Zhou, "Solution to generalized sylvester matrix equations," *Automatic Control, IEEE Transactions on*, vol. 53, no. 3, pp. 811 –815, april 2008.